Quantization Error in Time-Interleaved ADCs

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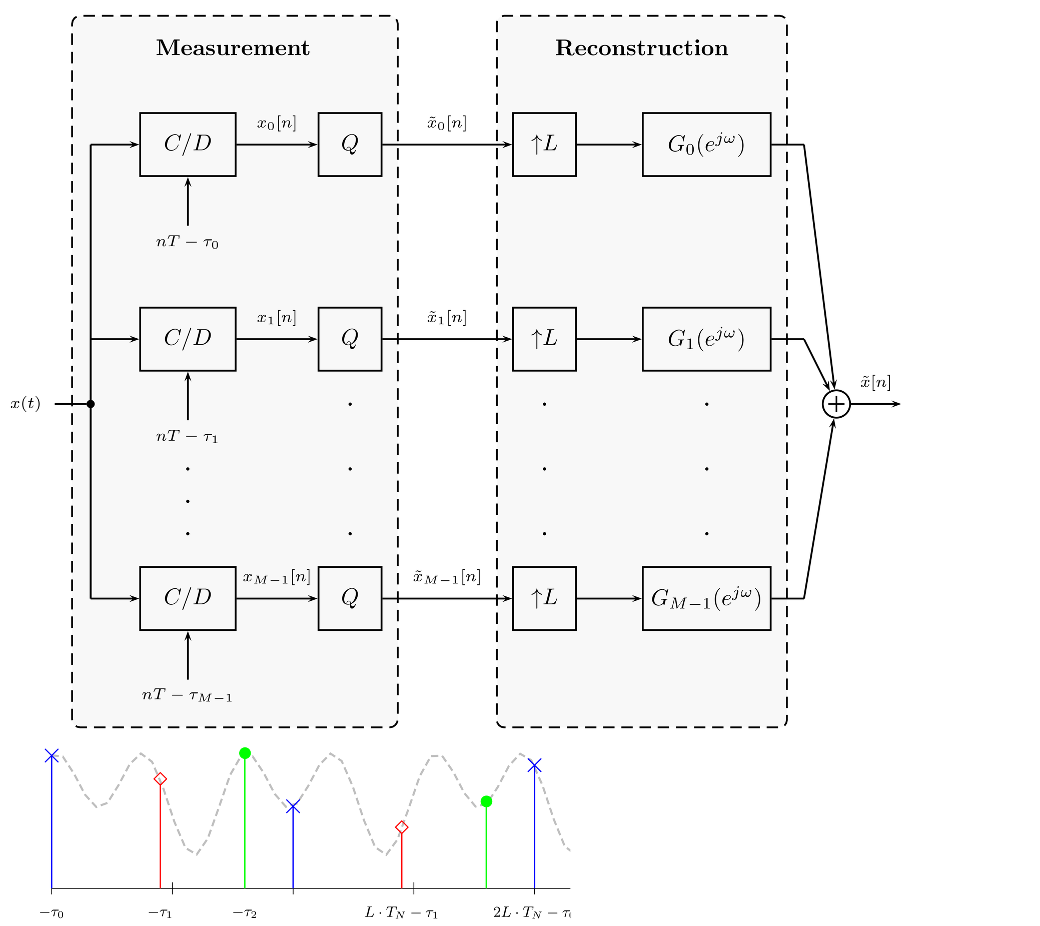
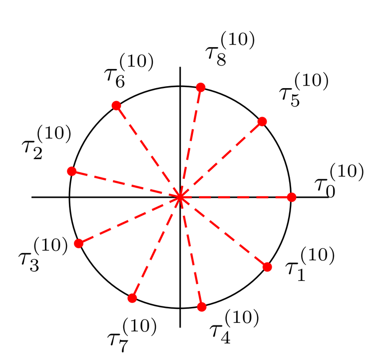
1. Motivation

- Time-interleaved ADCs are useful for high bandwidth signals or large oversampling ratios.
- They consist of M channels that uniformly sample an input at one L^{th} of the Nyquist rate.
- DT reconstruction filters derived in [1] and [2] minimize quantization error power σ_e^2.
- Optimize timing offsets τ_m and bit allocation b_m to minimize σ_e^2.

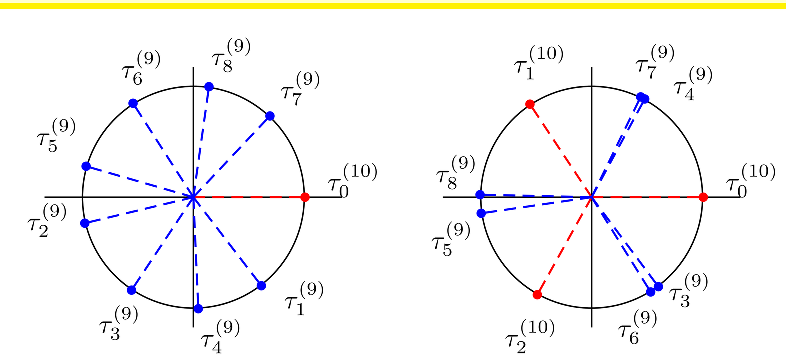


- Assume quantizer additive noise model holds.
- Homogeneous bit allocation $\longrightarrow$ uniform offsets

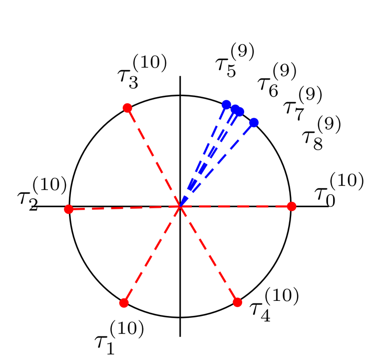


(a) Nine (10) bit quantizers

- Diverse bit allocation $\longrightarrow$ non-uniform offsets



(a) One (10) bit, Eight (9) bit quant. (b) Three (10) bit, Six (9) bit quant.

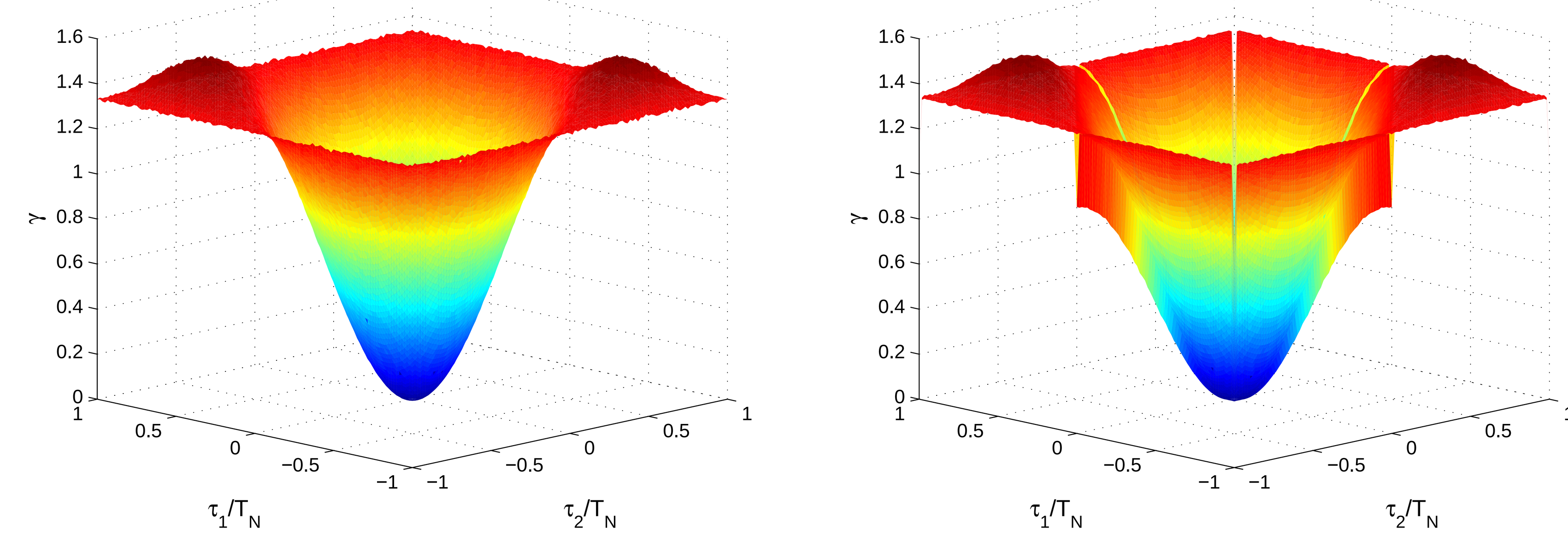


(c) Five (10) bit, Four (9) bit quant.

  - High-precision quant. $\leftrightarrow$ large time separation
  - Low-precision quant. $\leftrightarrow$ small time separation

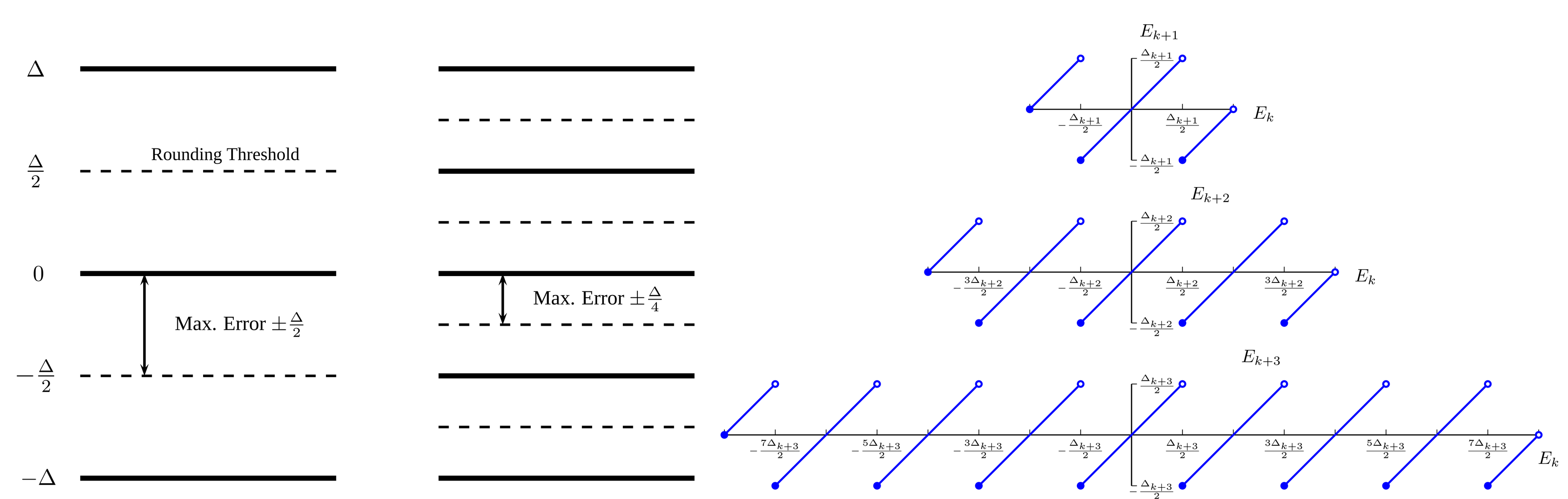
3. Validity of Additive Noise Model

- In practice, quantization error in different channels may be correlated.
  - Close timing offsets for identical quantizers $\longrightarrow$ degraded performance
  - Close timing offsets for different quantizers $\longrightarrow$ improved performance

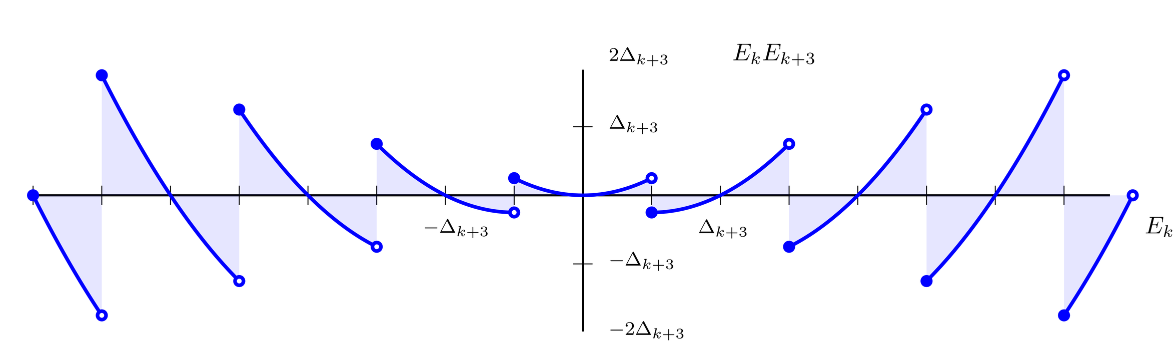


- Simulated reduction factor $\gamma = \frac{\sigma^2}{\sigma^2_{e_{min}}}$ for noise model (left) and uniform quantizers (right) with $M = 3$, $L = 2$, and $\tau_0 = 0$.

- Analyze two quantizers with k and $k+n$ bits operating on same analog sample.
- Deterministic relationship between quantizer error for different bits.



- Assume additive noise model holds for k bit quantizer.
- Covariance proportional to area under set of piecewise quadratic curves.



$$\mathrm{cov}(E_k, E_{k+n}) = \frac{1}{2^n \Delta_{k+n}} \int_{-2^{n-1}\Delta_{k+n}}^{2^{n-1}\Delta_{k+n}} e_k E_{k+n}(e_k)\, de_k \qquad (1)$$

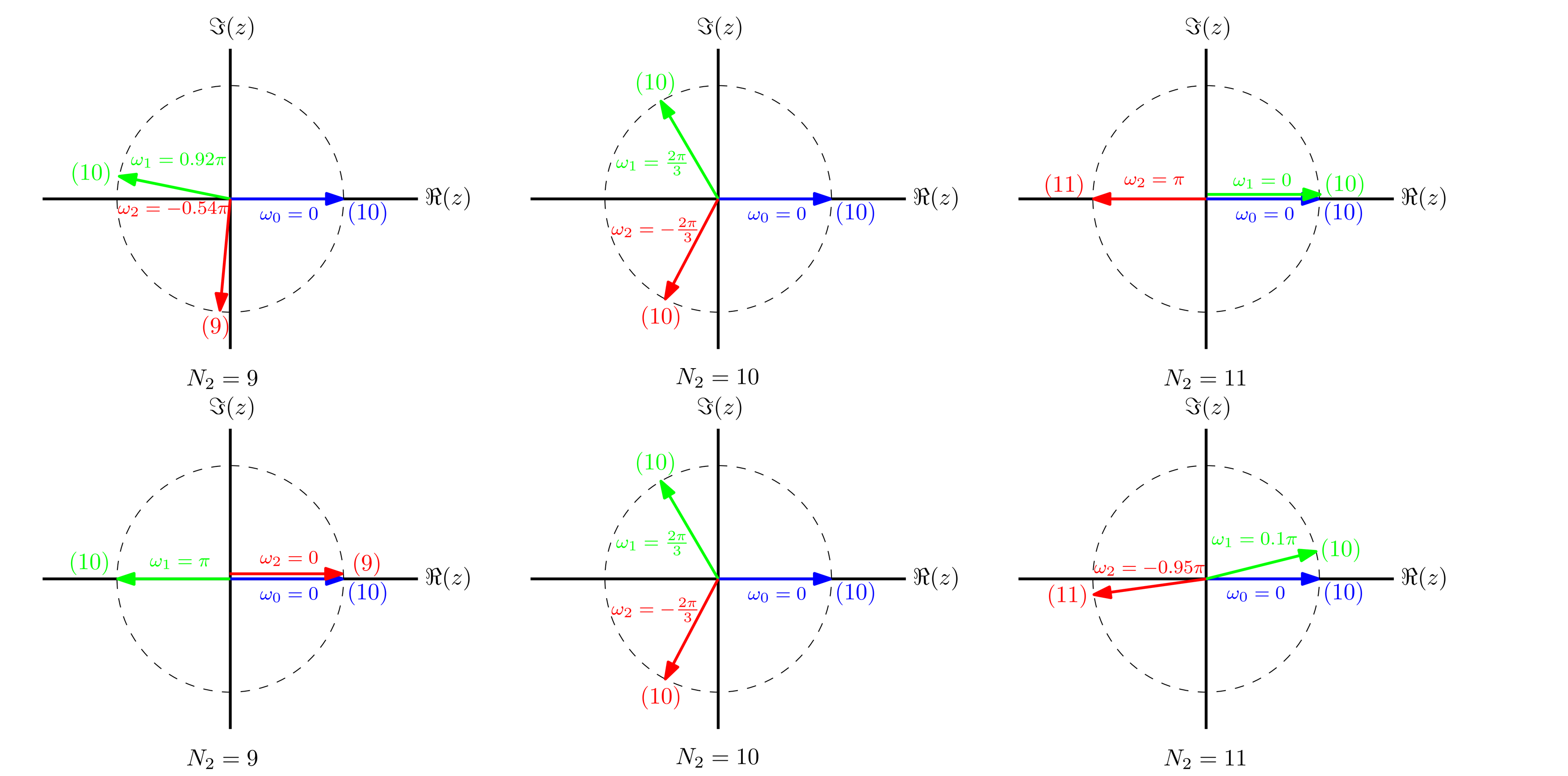
$$= \frac{\Delta^2_{k+n}}{12}\left(\overbrace{\frac{1}{2^n}}^{\text{base}} + \overbrace{\left(1 - \frac{1}{2^{n-1}}\right)}^{\text{middle}} + \overbrace{\left(1 - 3\cdot 2^{n-1}\right)\left(\frac{1}{2^n}\right)}^{\text{outermost}} \right) \qquad (2)$$

$$= (-1/2)\,\frac{\Delta^2_{k+n}}{12} \qquad (3)$$

$$\mathrm{cov}(E_k, E_{k+n}) = (-1/2)\,\mathrm{var}(E_{k+n})$$

5. Effect of Correlation

- Consider the case when $M = 3$, $L = 2$, and $\mathbf{b} = [10\ 10\ N_2]$.
- Optimal offsets for noise model (top) and quantizers (bottom) shown below.



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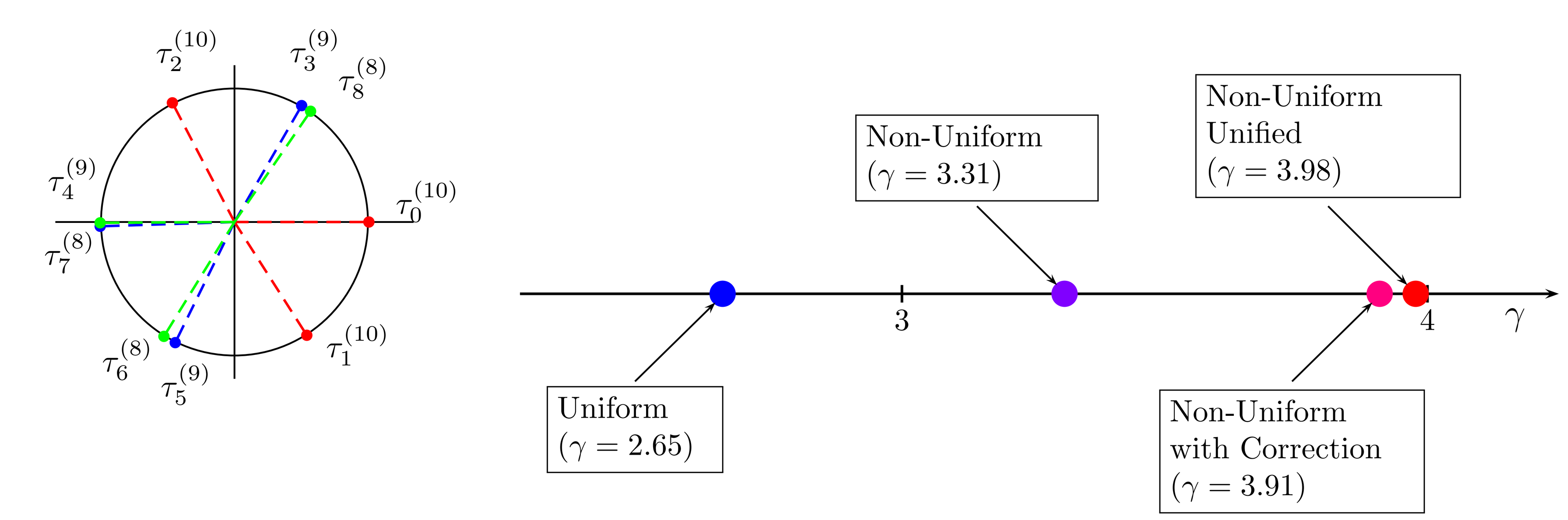
1. Optimize predicted timing offsets with additive noise model.
2. Separate identical quantizers in close proximity.
3. Unify different granularity quantizers in close proximity (WLS).

- Consider the case when $M = 9$, $L = 6$, and $\mathbf{b} = [10\ 10\ 10\ 9\ 9\ 9\ 8\ 8\ 8]$.
- Optimal timing offsets with additive noise model are shown below.



- Improvement of 16.8% in σ_e^2 vs. previously best known non-uniform offsets.
- Improvement of 33.4% in σ_e^2 vs. previously best known uniform offsets.

- Additive noise model matches quantizer error well in most situations.
- Exploiting cross-channel error correlation can improve performance.

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[1] S. Maymon and A.V. Oppenheim. Quantization and compensation in sampled interleaved multi-channel systems. In *ICASSP*, 2010.
[2] S. Maymon. Sampling and Quantization for Optimal Reconstruction. PhD thesis, Massachusetts Institute of Technology, 2011.